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V Semester B.A./B.Sc. Examination, December - 2019

(CBCS) (F+R) (2016-17 and Onwards)

MATHEMATICS - VI

Time : 3 Hours

Max. Marks : 70

Instruction : *Answer **all** questions.*

PART - A

Answer **any five** questions. **5x2=10**

1. (a) Write Euler's equation when the function 'f' is independent of x and y.

   (b) Find the curve $\int_0^1 \left[12xy+(y')^2\right] dx=0$ with $y(0)=3$, $y(1)=6$.

   (c) Find the function y which makes the integral $I=\int_{x_1}^{x_2}\left[1+xy'+x(y')^2\right] dx$ an extremum.

   (d) Evaluate $\int_C x\,dy-y\,dx$, where 'C' is a line $y=x^2$ from (0, 0) to (1, 1).

   (e) Evaluate $\int_0^2 \int_0^1 (x+y)\,dx\,dy$

   (f) Evaluate $\int_0^1 \int_0^1 \int_0^1 \left(x^2+y^2+z^2\right) dx\,dy\,dz$

   (g) State Gauss Divergence Theorem.

   (h) Write vector form of Green's Theorem.

P.T.O.

PART - B

2x10=20

Answer **two full** questions.

2. (a) Prove necessary condition for the integral $I=\int_{x_1}^{x_2} f(x,y,y')\,dx$, where $y(x_1)=y_1$ and $y(x_2)=y_2$ to be an extremum that $\frac{\partial f}{\partial y}-\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)=0$.

   (b) Find the extremal of the functional $I=\int_{x_1}^{x_2}\left(y^2+y'^2+2ye^x\right)dx$.

OR

3. (a) Show that an extremal of $\int_{x_1}^{x_2}\left(\frac{y'}{y}\right)^2 dx$ is expressible in the form $y=ae^{bx}$.

   (b) Solve the variational problem $\delta\int_1^2\left[x^2y'^2+2y(x+y)\right]dx=0$ with the conditions $y(1)=0=y(2)$.

4. (a) Find the shape of a chain which hangs under gravity between two fixed points.

   (b) Find the extremal of the functional $I=\int_0^{\pi}\left(y'^2-y^2\right)dx$ under the conditions $y=0$, $x=0$, $x=\pi$, $y=1$ subject to the condition $\int_0^{\pi} y\,dx=1$.

OR

5. (a) Find the extremal of the functional $\int_0^1\left(x+y+y'^2\right)dx=0$ under the conditions $y(0)=1$ and $y(1)=2$.

   (b) Find the geodesic on a right circular cylinder.

PART - C

Answer **two full** questions. **2x10=20**

6. (a) Evaluate $\int_C (x+2y)dx+(4-2x)dy$ along the curve C : $\frac{x^2}{16}+\frac{y^2}{9}=1$ in anticlockwise direction.

(b) Evaluate $\iint_R xy\,dx\,dy$ over the positive quadrant bounded by the circle $x^2+y^2=1$.

OR

7. (a) Evaluate $\int_C (x+y+z)ds$, where 'C' is the line joining the points (1, 2, 3) and (4, 5, 6) whose equations are $x=3t+1$, $y=3t+2$, $z=3t+3$.

(b) Change the order of integration and hence evaluate $\int_0^a \int_0^{2\sqrt{ax}} x^2\,dx\,dy$.

8. (a) Find the area $\iint_S \frac{y}{x} e^x\,dx\,dy$, where S is bounded by $x=y^2$ and $y=x^2$.

(b) Find the volume of the tetrahedron by the planes $x=0$, $y=0$, $z=0$ and $x+y+z=a$.

OR

9. (a) Change into polar co-ordinates and evaluate $\int_0^\infty \int_0^\infty e^{-\left(x^2+y^2\right)}dx\,dy$.

(b) If R is the region bounded by the planes $x=0$, $y=0$, $z=0$ and $x+y+z=1$. Show that $\iiint_R z\,dx\,dy\,dz=\frac{1}{24}$.

PART - D

Answer **two full** questions. **2x10=20**

10. (a) State and prove Green's theorem.

(b) Evaluate by Stoke's Theorem $\oint_C (yz\,dx + xz\,dy + xy\,dz)$, where C is the curve $x^2 + y^2 = 1$, $z = y^2$.

OR

11. (a) Verify Green's theorem $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where 'C' is the region bounded by parabolas $y^2 = x$ and $x^2 = y$.

(b) Using divergence theorem, show that :

(i) $\iint_S \vec{r} \cdot \hat{n}\,ds = 3V$ and

(ii) $\iint_S (\nabla r^2) \cdot \hat{n}\,ds = 6V$

12. (a) Evaluate using Gauss' divergence theorem $\iint_S \vec{F} \cdot \hat{n}\,ds$, where $\vec{F} = 2xy\hat{i} + yz^2\hat{j} + xz\hat{k}$ and S is the total surface of the rectangular parallelopiped bounded by the planes $x=0$, $y=0$, $z=0$, $x=1$, $y=2$, $z=3$.

(b) Evaluate $\iint_S \left(\text{Curl}\,\vec{F}\right) . \hat{n}\,ds$ by Stoke's theorem, where $\vec{F} = (y - z + 2)\hat{i} + (yz + 4)\hat{j} - xz\hat{k}$ and S is the surface of the cube $0 \le x \le 2$, $0 \le y \le 2$, $0 \le z \le 2$.

OR

13. (a) Evaluate $\iint_S \vec{F} \cdot \hat{n}\,ds$ using divergence theorem, where $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - xz)\hat{j} + (z^2 - xy)\hat{k}$ taken over rectangular box $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$.

(b) Evaluate by Stoke's theorem $\oint_C (\sin z\,dx - \cos x\,dy + \sin y\,dz)$, where C is the boundary of rectangle $0 \le x \le \pi$, $0 \le y \le 1$, $z = 3$.

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